COMPARISON OF METHODS FOR INDICATING THE MEASUREMENT UNCERTAINTY OF INTEGRAL PARAMETERS ON THE BASIS OF SPECTRAL DATA BY MEANS OF THE MEASUREMENT UNCERTAINTY OF THE f_1' VALUE

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ABSTRACT

In their paper, the authors compare the calculation of the measurement uncertainty of the f_1' value, with calculations made using two different approaches GUM [1] and GUM Supp. 1 [2].

The f_1' value was used as an example for the integral parameter on the basis of spectral data. By means of the $V(\lambda)$ function, this parameter, which has remained very popular in the quality assessment of measuring devices used in photometry, is determined using a non-linear evaluation function.

Keywords: f_1', measurement uncertainty, Monte Carlo simulation

INTRODUCTION

Dealing with problems in the range of photopic filter design for imaging luminance measurement devices (ILMDs), the first author makes calculations for the f_1' value depending as well on different light angles over the image (a specific problem of ILMDs without telecentric lenses) as on the measurement uncertainty of glass transmissions and manufacturing tolerances influencing the glass transmission [3]. Other authors calculated the f_1' value depending on the image position or the angle over the image for ILMDs [4].

Beside these difficulties it is not easy to calculate the measurement uncertainty of the f_1' value based on spectral measurements. To check the possibilities in this field the authors carried out different calculations explained in the following paragraphs.

BASICS

The f_1' value is defined in CIE Pub. 69 [5]. Starting from the spectral sensitivity $s(\lambda)$ of a photometer, the special standardized sensitivity $s^*_{\mathrm{rel}}(\lambda)$ is calculated:

$$s^*_{\mathrm{rel}}(\lambda) = \frac{\int S_{\lambda,NLA}(\lambda)\cdot V(\lambda)d\lambda}{\int S_{\lambda,NLA}(\lambda)\cdot s(\lambda)d\lambda}\cdot s(\lambda) \qquad (1)$$

$S_{\lambda,\mathrm{NLA}}(\lambda)$ … Standard illuminant A

Based on the $s^*_{\mathrm{rel}}(\lambda)$ sensitivity, the f_1' value is calculated as follows:

$$f_1' = \frac{\int \left| s^*_{\mathrm{rel}}(\lambda) - V(\lambda)\right| \mathrm{d}\lambda}{\int V(\lambda)\mathrm{d}\lambda}\cdot 100\% \qquad (2)$$

CALCULATION OF MEASUREMENT UNCERTAINTY OF f_1' VALUE

In the following paragraph, two approaches estimating the measurement uncertainty of f_1', are compared. The first approach is based on the Taylor series approximation [1]. The second approach uses the Monte Carlo Simulation [2].

TAYLOR SERIES

According to a method described in [6] using the methods of GUM [1], one can estimate the measurement uncertainty of a value $q = f\{s(\lambda)\}$ using the (numerical) partial derivative of *q* at different wavelengths. The basis for this approach is a segmentation of the spectral data in selected wavelength sections for which the partial derivative of *q* can be calculated numerically.

$$q = f\{s(\lambda)\} = f\{s(\lambda_1), s(\lambda_2), ..., s(\lambda_n)\} \quad (3)$$

$$\frac{\partial q}{\partial s(\lambda_i)} = \frac{f\{s(\lambda_1), s(\lambda_2), ..., s(\lambda_i) + \Delta s, ..., s(\lambda_n)\} - f\{s(\lambda)\}}{\Delta s}; i = 1, ..., n \quad (4)$$

$\Delta s(\lambda_i)$ is used so that we can justify a linear approximation around the sampling point.

The combined measurement uncertainty can be calculated with:

$$u(q) = \sqrt{\sum_{i=1}^{n} \left\{ \frac{\partial q}{\partial s(\lambda_i)} \right\}^2 u^2\{s(\lambda_i)\}} \quad (5)$$

using the standard measurement uncertainty $u\{s(\lambda_i)\}$ of $s(\lambda_i)$ at the sampling point λ_i and the assumption of non correlated measurements at different spectral sampling points.

At this point of modeling it has to be checked whether and in how far a linearized approach is justified. The following weak points are identifiable:

1. For small values of the $V(\lambda)$ function in the border area of the visual spectral range, the values of the $V(\lambda)$ function are possibly smaller than the assigned measurement uncertainties. Therefore, a “negative” sensitivity is used – which is physically absurd. For the numerical calculations only positive values $\Delta s(\lambda_i)$ are used, but the effect in the negative range cannot be fully specified. The clipping of the spectral sensitivities to positive values is physically correct. However, the influence of the probability distribution of the measurement values is enormous.

2. With the spectral sensitivity used through the special standardization in (1) the sign of the difference $s^*_{\mathrm{rel}}(\lambda) - V(\lambda)$ is changed very often. At this point, (4) is not valid because the function is not continuously differentiable. For this reason, a linearized model cannot be employed. Here, one can only work with mathematics for non-linear models.

3. The f_1' is zero for the ideal case. Each deviation increases f_1', just as each additional deviation resulting from measurement uncertainties does. In this case, not only the measurement uncertainty is wrongly estimated, also the expected value for f_1' increases with increasing measurement uncertainties for the values at spectral sampling points.

This conclusion is very important. By means of modified assumptions for the measurement uncertainty both the measurement uncertainty of the integral value **and** the expected value of the measurement have been changed.

MONTE CARLO SIMULATIONS

With the expansion of the GUM [1], through the first supplement [2] one can use standardized methods to calculate the measurement uncertainties with Monte Carlo simulations.

Applying this method, the input variables X_i are described with their PDF (probability distribution function) and the model $Y = f(X_i)$ has to be implemented in the simulation program. During simulation, a random number generator calculates the realization for the random variables (random samples) required, and for every input value the simulation program calculates the model output. Through the monitoring of the model output the software calculates an estimation of the output PDF and all related statistical data (mean, variance, …).

No complex programs need to be used for simulation. Wanting to carry out simulation in an easy way, the user may employ Excel AddIns [7].

EXAMPLES

Below, the effects explained above are demonstrated on the basis of some examples. All MonteCarlo simulations had been carried out with Excel™ or Mathematica™ with 100000 or 200000 trails.

Comparison of the Taylor and the Monte Carlo simulation

Assuming one gets, during the measurement of an ideally matched sensor for every wavelength sampling point, the expected value (the V(λ)-function), then one obtains $f_1' = 0\,\%$ according to the first method, i.e. the value of an ideally matched sensor.

With a standard measurement uncertainty[1] of $u\{s(\lambda_i)\} = 0.005$ for every wavelength sampling point the values shown in the table below are obtained. Furthermore, we illustrate the simulation results for a "real" sensor, with the same assumption for the measurement uncertainty being made.

	$f_1'(\%)$	$u_c(f_1')$
Ideal Sensor		
Taylor	0.00	0.48
MonteCarlo	**1.54**	**0.14**
Real Sensor		
Taylor	2.48	0.42
MonteCarlo	**4.11**	**0.29**

Table 1: Comparison with between Taylor and Monte Carlo for standard simulation

For both cases, the estimation of the measurement uncertainty using the Monte Carlo simulation is significantly smaller. The estimated mean value, however is significantly increased if we estimate both values the mean value **and** the measurement uncertainty with the Monte Carlo simulation.

In the following figures, the distributions of the f_1' values are displayed for an ideally matched sensor and a real sensor.

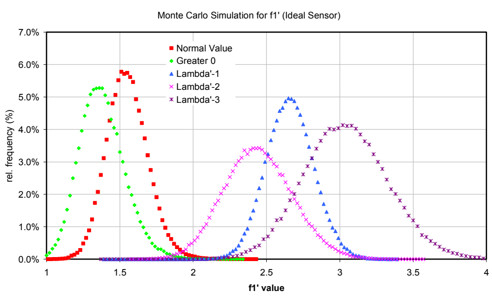

Figure 1: MC simulation "Ideal Sensor"

[1] This is not very realistic and serves only demonstration purposes.

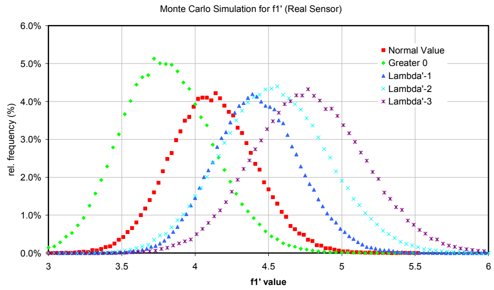

Figure 2: MC simulation "Real Sensor"

Each figure shows five curves:

Normal Value	Normal simulation
Greater0	Simulation with clipping $s(\lambda_i) \geq 0$
Lambda' (1-3)	Simulation like Normal with additional consideration of wavelength errors, see **Table 3**

Effect of clipping

The fist weak point for the Taylor Method was the clipping effect, in the case that the measurement uncertainty is greater than the $V(\lambda)$ function. By means of the Monte Carlo Method we can show the following:

	$f_1'(\%)$	$u_c(f_1')$
Ideal Sensor	1.37	0.15
Real Sensor	3.79	0.32

Table 2: Monte Carlo simulation for Ideal and Real Sensor with clipping effect

Due to the clipping the mean value is reduced because of smaller differences to the $V(\lambda)$ function in the clipped values. The expected measurement uncertainty is approximately the same.

The question is: How to deal with the clipping effect during simulations?

Effect of wavelength errors

The effects resulting from wavelength errors are very difficult to estimate by means of the Taylor method. With the Monte Carlo simulation one can generate a new wavelength, e.g. as an offset value:

$$\lambda' = \lambda + NPD(\lambda_{mean}, \lambda_{var}) \qquad (6)$$

The calculation works with the measurement values at the selected wavelength sampling points λ_i and the standard values coming form the sampling points $\lambda_i \left(V(\lambda'), S_A(\lambda')\right)$. For Eq. (6) other models are possible.

Simulation results for simple examples:

Ideal Sensor $\lambda_{mean}, \lambda_{\text{var}}$	$f_1'(\%)$	$u_c(f_1')$
1:1 nm, 0 nm	2.65	0.16
2:0 nm, 1 nm	2.43	0.23
3:1 nm, 1 nm	3.03	0.29

Table 3: Comparison of Monte Carlo simulation results with different wavelength errors

As displayed in **Figure 1** and **Table 3** the uncertainty for the wavelength sampling points is very important.

SUMMARY

To summarize, it can be stated that - for estimating the measurement uncertainty of the f_1' value - a linearized approach according to [6] can no longer be used for well matched photometers. In this case, methods described in [2] using the Monte Carlo simulation must be applied.

The advantage of these methods is that they allow also non-linear models to be included, with no analytical preliminary work being required. Furthermore, it would be possible to include in a very simple way the influence effected by the measurement uncertainty on the determination of the wavelength in the calculation. However, the only potential disadvantage could be the long computing time necessary for the simulation. The simulation approach is explained with simple Excel sheets.

In an ideal case (ideally matched photometer), the f_1' value is equal to zero. Each deviation increases the value, thus, also each (additional) deviation resulting from measurement uncertainty. In other words, the value of the parameter (expected value) increases with increasing measurement uncertainty, which is an essential fact. Due to changing assumptions for the measurement uncertainties of the partial quantities measured, not only the measurement uncertainty of the result value changes, but also the result value of the measurement itself.

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